Lecture notes on risk management, public policy, and the financial system

Securitization and structured credit risk

Allan M. Malz

Columbia University

What is securitization?

- Closely-related terms for creation of securities backed by pools of financial assets:

  Securitization generally refers to cash securities backed by mortgages, consumer debt and leases

  Structured credit or **finance** generally refer to securities backed by bank debt or bonds, or securitization in **synthetic** credit derivatives form
- Resulting securities called **asset-backed securities** (ABS)
  - If backed by residential or commercial mortgage loans, generally called **mortgage-backed securities** (MBS)
- **Collateralized debt obligations** (CDOs) are securitizations in which the asset pool consists of bank loans or other securitizations

Essential functions of securitization

Pooling of risk and diversification, similar to banks and mutual funds

Risk transfer: separation of **loan origination** from balance-sheet investment/use of capital

- **Originate-to-distribute** model: loan issuance based on likelihood of securitization "exit," return of capital to originator
- Shifts banks' revenue source from net interest margin to fees
- Loan origination is primarily information creation: selecting borrowers

Risk distribution: creation of securities with different risk-reward characteristics

- Tranched products may have very different default characteristics from underlying loan or asset pool

Regulatory optimization: originators reduce required equity funding for a given level of asset risk

- Taking →**risk retention** into account

Asset types in collateral pools

Existing loans may be sold into collateral pools

- Examples include residential and commercial mortgages, bank loans
- Residential mortgage pools may be highly granular
- Pools consisting of commercial mortgages generally less granular and may consist of only a few loans

Revolving pools of primarily short-term debt, e.g. credit card receivables and auto loans

- Securitization begins with initial pool that is replaced as it is repaid by fresh debt

Future flows of assets, such as remittances from abroad

- Revolving pools and pools consisting of future flows tend to be highly granular

Risk types in collateral pools

Prepayment risk: risk of early payment of pool loan principal, leading to

- Cash flows occurring earlier than anticipated
- Shortening of duration of loans
- Possible need to reinvest funds at lower interest rates

Credit risk: risk of default of pool loans

Participants in a securitization

Originators: initial lenders in collateral pool debt instruments

Sponsor organizes and structures the securitization

- May be same entity as originator

Servicer collects principal and interest from collateral and distributes to liabilities

- May carry out credit monitoring

Investors in the liabilities of a securitization

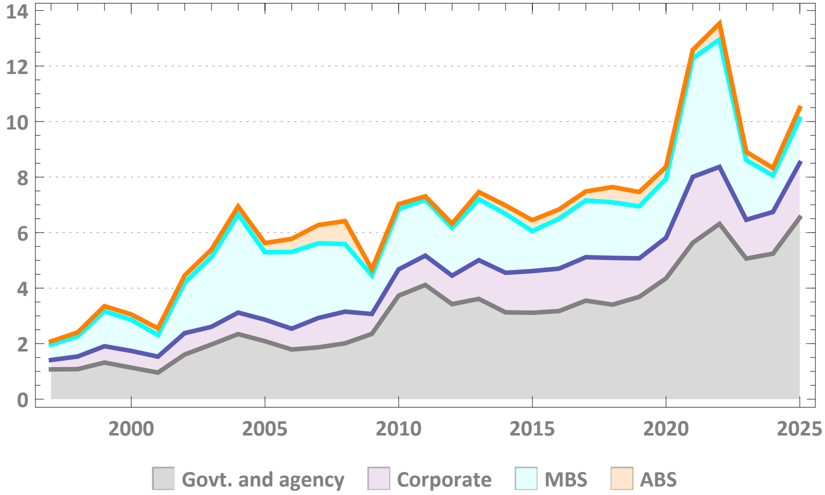
Capital structure

- **Special purpose vehicle** (SPV) owns **collateral pool** or underlying assets, issues debt
  - Must follow a **true sale** of underlying assets
  - But **implicit recourse** to or guarantee by seller of underlying assets may remain
- Generally several bonds or **tranches** in (generally) clearly defined priority, with equity tranche at bottom.
- Intermediate subordinated tranches called **mezzanine**
- Bonds may suffer **material impairment** rather than default: missed interest payments, deterioration of collateral pool performance
  - If "thin"$\rightarrow$binary risk (see also default correlation)
- **Overcollateralization** creates protection for tranches higher senior to equity
  - Collateral pool larger than volume of bonds issued
  - Loans in pool generally also overcollateralized
- Equity most highly leveraged vis-à-vis collateral pool, senior least leveraged

U.S. securitization trends

- **Mortgage-backed securities** (MBS) by far the largest segment of securitized debt market
  - Share of total U.S. bond issuance declining since crisis from nearly half in 2005
- Non-mortgage **asset-backed securities** (ABS) issued since 1985 in U.S.
- Tranched products issued since early 1980's
  - Introduced in the form of **collateralized mortgage obligations** (CMOs), protecting against prepayment risk
  - Tranching used in most ABS to protect against credit risk
- Early on, most issuance in auto loan and credit-card receivables segments
- Subsequently, growth in CDOs
- Rapid recent growth in **collateralized loan obligations** (CLOs)

Bond issuance in the U.S. 1996–2024



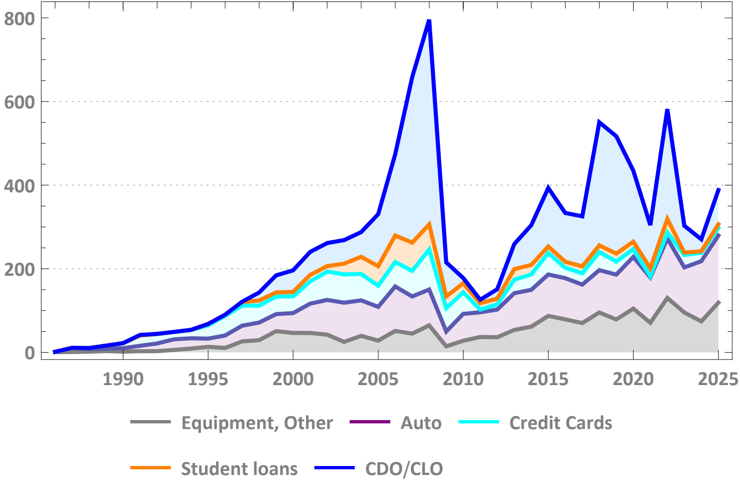
Issuance in $ trillions, annual. *Source*: SIFMA, U.S. Bond Market Issuance and Outstanding.

CDO issuance after the crisis

- Large volume of CDO issuance precrisis, but low issuance immediately following crisis
  - CDOs long-lived, esp. legacy CDOs with credit problems
  - →Much smaller decline in share of outstanding
- **Leveraged loans:** typically
  - Defined as large loans to sub-investment grade firms
  - Floating-rate loans with wide spread to index rate
  - **Syndicated:** issued by several banks, each bearing risk only of own issuance
  - Intended for sale into CLO asset pool

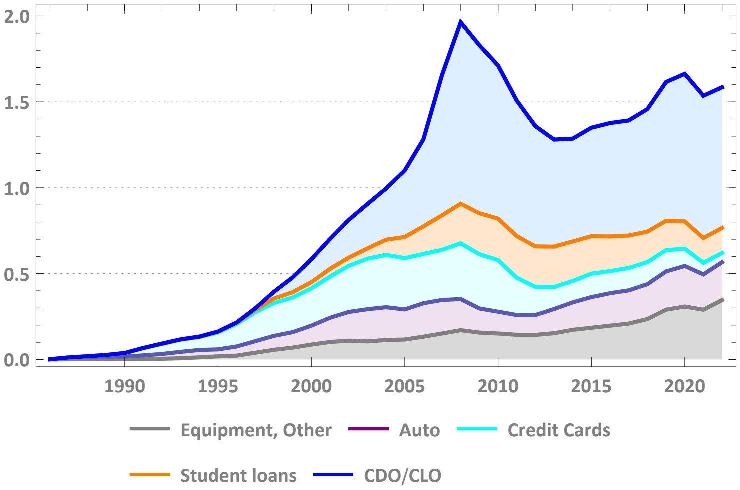
"Here are my notes on collateralized debt obligation. Make me sound like I know what I'm talking about."

ABS issuance in the U.S. 1985–2024



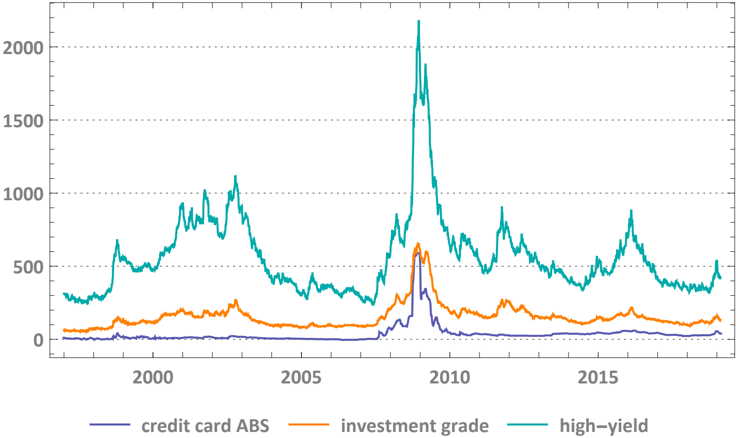
Issuance in $ billions, annual. CLOs includes CBOs and other CDOs. *Source*: SIFMA, U.S. ABS Issuance and Outstanding.

ABS outstanding in the U.S. 1985–2021



Outstanding volume in $ billions, annual. CLOs includes CBOs and other CDOs.
Source: SIFMA, U.S. ABS Issuance and Outstanding.

U.S. credit spreads 1997–2019



Option-adjusted spreads (OAS) over swaps, in basis points. Credit card ABS: 5-year AAA U.S. credit-card ABS, weekly 27Dec1996 to 21Feb2019. Investment grade and high yield: BofA Merrill Lynch U.S. Corporate Master OAS indexes (C0A0 and H0A0). *Sources*: Barclays, FRED.

Waterfall

- **Waterfall:** set of rules about how collateral cash flows and losses distributed to tranches
  - Cash flows distributed "top-down," to senior tranches first, then mezzanine, residual to equity
  - Losses distributed "bottom-up," equity written down first
- **Attachment** and **detachment points:**
  - Attachment point of a tranche is the fraction of pool losses to which it is *not* exposed
  - Attachment point of a tranche is also the fraction of total liabilities subordinate to it
  - Detachment point of a tranche is the fraction of pool losses at which it is entirely wiped out
  - Attachment point of one tranche is the detachment point of the next-most junior tranche
  - The difference between the detachment and attachment points of a tranche equals its **thickness**, or share of total liabilities
- Exceptions can be written into operating agreements

Example of a securitization

Assets	Liabilities
Underlying debt instruments:	Equity note \$5 mill.
	Mezzanine debt \$15 mill. Coupon: risk-free+500 bps
\$100 mill. of loans Rate: risk-free+750 bps	Senior debt \$80 mill. Coupon: risk-free+100 bps

Parameters for the example:

Risk-free rate (%)	r^f	3.5
Loan interest rate (%)	r_l	11.0
Mezzanine coupon (%)	c_m	8.5
Senior coupon (%)	c_s	4.5
Mezzanine attachment point (% of liabilities)	a_m	5
Senior attachment point (% of liabilities)	a_s	20

Risk assumptions in the example

- Collateral pool:
  - One-year loans, no prepayment
  - Expected default rate $\pi = 0.05$ (5%)
  - Expected recovery 0
- Similar to typical **subprime auto loan** securitizations
  - Granular collateral pool
  - Debt has short **weighted-average life** (WAL)
  - Fairly high default rate
- Liabilities:
  - One-year annual coupon bonds
  - Equivalent to zero coupon bonds (but issued at par, not at discount)

Credit enhancement and pricing in the example

- Senior bond (“20–100”): $20 mill. equity note plus mezzanine debt
    - Senior bond has priority claim over mezzanine
    - Both bonds have priority claim over equity
- Mezzanine bond (“5–20”): $5 mill. equity note
- In addition to any overcollateralization of the underlying loans
- Credit enhancement of senior and mezzanine bonds assumed sufficient to price them at par on issuance
    - ⇔Spreads are sufficient compensation for credit, other risks
- Equity note (“0–5”) assumed to price at par on issuance
    - Expected return 11.5%, i.e. if expected default rate realized

Structure of a securitization: stipulated cash flows

- Contactually-stipulated cash flows: principal and interest (P&I)
  - Due from loan obligor to SPV
  - Due from SPV to bonds
- Contactually-stipulated cash flows actually occur if no default or bond impairment
- Contactually-stipulated cash flows from underlying collateral pool into SPV
  - Each obligor to pay P&I of $1 + r_l \times$ loan principal in one year
  - Aggregate for pool: $1 + r_l \times$ total par value of collateral
- Contactually-stipulated cash flows to bondholders
  - SPV to pay P&I of $1 + c_s$ or $1 + c_m \times$ bond principal in one year
- *No* contractually-stipulated cash flows due to *equity* tranche

Stipulated cash flows to bondholders in the example

Senior bond to receive par value of 80 percent of pool principal plus coupon in one year:

$$(1 - a_s)(1 + c_s) \times \text{total par value of collateral}$$

Mezzanine bond to receive par value of 15 percent of pool principal plus coupon in one year:

$$(a_s - a_m)(1 + c_m) \times \text{total par value of collateral}$$

	senior	mezzanine
Tranche thickness (% of SPV liabilities)	0.80	0.15
P&I due (% of pool principal)	83.600	16.275
P&I due ($)	83 600 000	16 275 000

Recent examples

- Vehicle for **multilateral development bank** (MDB) to
  - Increase lending without growing balance sheet
  - Involve private sector financial intermediaries in emerging market business lending
- Early example 2018: originator/sponsor African Development Bank (AfDB) securitisation, dubbed "Room2Run"
  - **Synthetic securitisation transaction** (SST)
- 2024 example: originator/sponsor IDB Invest, subsidiary of Inter-American Development Bank (IDB)

  Assets: \$1 billion of IDB Invest project finance loans

  Liabilities/tranches: **Senior tranche:** \$870 million

  Mezzanine tranche: \$100 million, part sold to investor Newmarket Capital, the remainder guaranteed by insurers AXIS and AXA

  Junior tranche: \$30 million, retained by IDB Invest

The 2025 International Finance Corporation CLO

- Originator/sponsor: **International Finance Corporation** (IFC), member of the World Bank Group
  - Collateralized loan obligation (CLO)
  - SPV name: IFC Emerging Markets Securitization 2025-1, Ltd
- Part of IFC Emerging Markets Securitization Program (EMSP)
  - Facilitate emergence of originate-to-distribute channel for emerging market debt
- Important features
  - Aaa-rated (Moody's) senior tranche
  - True sale rather than synthetic securitisation
  - Institutional investors in senior tranche

Balance sheet of the securitization

Assets: \$510 million of senior secured and unsecured loans

Liabilities/tranches: **Senior tranche:** \$320 million 10-year floating rate note

- Coupon SOFR+130bps
- Sold to private investors rather than retained by sponsor

Mezzanine tranche: \$130 million

- Coupon SOFR+320bps
- Retained by IFC, but insured by a consortium of credit insurers

Equity tranche: \$60 million, retained by IFC

Collateral cash flows under loss scenarios

- Waterfall begins with *underlying collateral*
- Loan proceeds (as multiple of notional) depend on default rate x:

$$\text{loan proceeds}(x) = (1-x)(1+r_l), \qquad 0 \le x \le 1$$

Waterfall and cash flows under loss scenarios: tranches

Senior bond has priority claim over mezzanine and equity

- Receives all loan proceeds up to its own par value and coupon $(1 - a_s)(1 + c_s)$:

$$\text{senior cash flow}(x) = \min[(1 - x)(1 + r_l), (1 - a_s)(1 + c_s)]$$

Mezzanine bond paid only if senior bond paid in full

- Receives all post-senior loan proceeds up to its par value and coupon $(a_s - a_m)(1 + c_m)$:

$$\text{mezzanine cash flow}(x)$$
$$= \max[\min[(1 - x)(1 + r_l) - (1 - a_s)(1 + c_s), (a_s - a_m)(1 + c_m)], 0]$$

Equity note receives remainder, if positive

$$\text{equity cash flow}(x)$$
$$= \max[(1 - x)(1 + r_l) - (1 - a_s)(1 + c_s) - (a_s - a_m)(1 + c_m), 0]$$

Tranche returns under loss scenarios

- Compute tranche returns as a function of loan default rate from tranche cash flows and "thickness"

$$\text{senior return}(x) = \frac{\text{senior cash flow}(x)}{1 - a_s} - 1$$

$$\text{mezzanine return}(x) = \frac{\text{mezzanine cash flow}(x)}{a_s - a_m} - 1$$

$$\text{equity return}(x) = \frac{\text{equity cash flow}(x)}{a_m} - 1$$

- We can measure the return to each tranche in stress scenarios
  - E.g. pool loss rate far in excess of expected default rate x
  - Maximum return if pool loss rate zero

	senior	mezzanine	equity
Baseline ($x = 0.05$) cash flow ($ mio.)	83.600	16.275	5.575
Baseline ($x = 0.05$) return (%)	4.50	8.50	11.50
Maximum ($x = 0$) return (%)	4.50	8.50	122.50
Stress case ($x = 0.125$) return (%)	4.50	−9.83	−100.00

Tranche returns are option-like

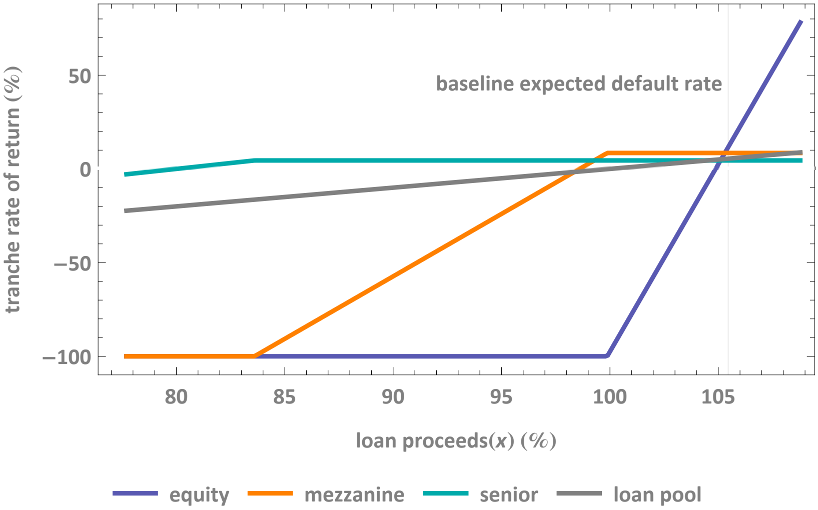
- Securitization tranches behave like options on underlying loan pool credit losses/proceeds
- Strike levels: attachment/detachment points

  Senior tranche behaves like a "short call" on loan pool proceeds
  Mezzanine tranche behaves like a "collar" on loan pool proceeds
  Equity tranche behaves like a "long put" on loan pool proceeds
- Payoff profiles and exercise prices defined in terms of loss levels at which the bond tranches default
- Leads to market risk behavior driven by changes in expected default rate and default correlation

Tranche thickness and leverage

- Junior securitization tranches (mezzanine and equity) contain **embedded leverage**
- Thin tranches take proportionally greater losses for a given pool loss rate
- Tranche suffers losses only once its attachment point is breached
- Embedded leverage thus generated by two characteristics
  - Tranche thinness in conjunction with
  - Low position in waterfall
- For example, a 10 percent loan default rate
  - Barely brings pool rate of return to zero
  - But leads to total loss on equity tranche

Pool and tranche returns in the example



Default of a bond tranche

- Event of default of a bond tranche defined similarly to non-securitization bond: failure to pay principal or interest due
  - In our example, bond default occurs only at single one-year payment date for bond principal and interest
  - Insolvency may become evident well within one year, e.g. if realized loan defaults high
- Find default-triggering loss level x°:

$$\left.\begin{array}{r}\text{senior}\\ \text{mezzanine}\end{array}\right\}\text{defaults}$$

$$\Leftrightarrow (1-x)(1+r_l) < \left\{\begin{array}{l}(1-a_s)(1+c_s)\\ (1-a_s)(1+c_s)+(a_s-a_m)(1+c_m)\end{array}\right.$$

  - Tranche with default-triggering loss level x° defaults if $\tilde{x} \geq x^\circ$
  - Probability of tranche default is $\mathbf{P}\left[\tilde{x} \geq x^\circ\right]$
- If senior bond defaults, mezzanine bond must also default
- Equity tranche cannot default
  - But can suffer lower-than-expected/negative returns

Default and distressed returns in the example

- Default-triggering loss level x° for bond tranches:

$$x^\circ = \left\{ \begin{array}{c} 1 - \dfrac{(1-a_s)(1+c_s)}{1+r_l} \\ 1 - \dfrac{(1-a_s)(1+c_s)+(a_s-a_m)(1+c_m)}{1+r_l} \end{array} \right\}$$

$$\text{for the} \left\{ \begin{array}{c} \text{senior} \\ \text{mezzanine} \end{array} \right\} \text{tranche}$$

	senior	mezzanine
x° (%)	24.685	10.023

- Loss level x at which equity tranche return is zero:

$$1 - \frac{(1-a_s)(1+c_s)+(a_s-a_m)(1+c_m)+a_m}{1+r_l}$$

or 5.518 percent

  - Loss level at which equity tranche is wiped out (return $= -100$ percent) is identical to mezzanine default-triggering loss level

Valuation and risk modeling approaches

- Risk analysis of securitization tranches based on risk analysis of underlying loan pool
- Typical rating agency approach: credit stress scenarios
    - "What-if" scenarios featuring much higher-than-expected default rates
    - Stipulate default and recovery behavior of the loan pool over time
    - Trace through cash flow results and effects on each tranche
    - Determine loss levels that "break" each tranche
- Formal credit portfolio modeling approaches
    - Simulation approaches, generally using (→)copula models
    - Specific credit models, e.g. single-factor model

Credit risk analysis of a securitization

- Assume collateral pool highly granular
- Combine risk analysis of loan pool with securitization waterfall to analyze credit risk of any securitization tranche
- Risk analysis based on credit loss distribution of tranches
- Explore impact of change in default probability and of high correlation
- No zero recovery case for liability structure as a whole or for individual tranches
    - "Material impairment"
    - Even if recovery 0 for underlying loans
    - Apart from degenerate cases ($\pi = 0$ or a detachment point),

Applying single-factor model to example

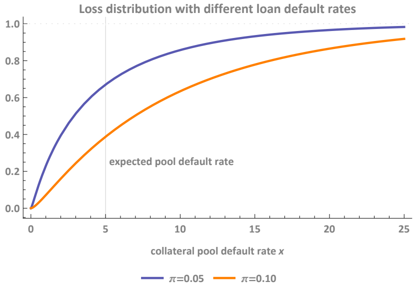
- Distributions of pool losses/tranche returns depend on market factor
    - Given assumptions on expected default rate (π) and correlation to market factor (β)
    - Default correlation among loans is β^2
- Cumulative distribution function of pool losses (a random variable $\tilde{x}$) in single-factor model:

$$\mathbf{P}\left[\tilde{x} \leq x\right] = \Phi\left[\frac{\sqrt{1-\beta^2}\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\beta}\right]$$

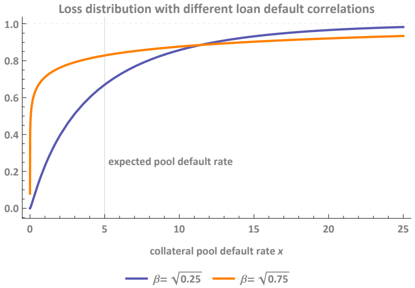
- Assume securitization constructed under baseline parameters
- Study effect on credit loss distributions of varying π and β
- Baseline parameters:

$$\pi = 0.05$$
$$\beta = \sqrt{0.25}$$

Cumulative distribution function of pool losses



Both plots use baseline parameter $\beta = \sqrt{0.25}$.



Both plots use baseline parameter $\pi = 0.05$.

Correlation and collateral pool losses

- Default correlation has large impact on risk of equity and senior tranches
- Higher default correlation→higher likelihood of default clusters
- Loss distribution becomes skewed
- →Higher tail risk, i.e. likelihood of *both*
  - Very large losses
  - Very small losses

	Correlation assumptions	
	moderate	high
	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.75}$
$\mathbf{P}[\tilde{x} \leq 0.01]$	0.230	0.711
$\mathbf{P}[\tilde{x} \geq 0.25]$	0.017	0.066

Tranche risk analysis

- Each scenario/realization x of pool defaults has
  - Probability $\mathbf{P}[\tilde{x} \leq x]$
  - Waterfall→cash-flow consequences for each tranche
- →Cumulative distribution function of cash flows for each tranche
  - For example, cash flow CDF of the senior tranche is the set of pairs

$$\{\text{senior cash flow}(x), \mathbf{P}[\tilde{x} \leq x]\}, \quad x \in [0,1]$$

  - Can be computed for all tranches
- Can be mapped into CDF of returns as well as cash flows for each tranche

Probability of default of a bond tranche

- Probability of default of a bond tranche can be computed via loss distribution function
- We have calculated default-triggering loss level for each bond tranche based on its coupon and the waterfall
- Tranche with default-triggering loss level x° defaults if pool losses reach or exceed that level: $\Leftrightarrow \tilde{x} \geq x^\circ$
- $\Rightarrow$Probability of tranche default is

$$\mathbf{P}\left[\tilde{x} \geq x^\circ\right] = 1 - \mathbf{P}\left[\tilde{x} \leq x^\circ\right] = 1 - \Phi\left[\frac{\sqrt{1-\beta^2}\Phi^{-1}(x^\circ) - \Phi^{-1}(\pi)}{\beta}\right]$$

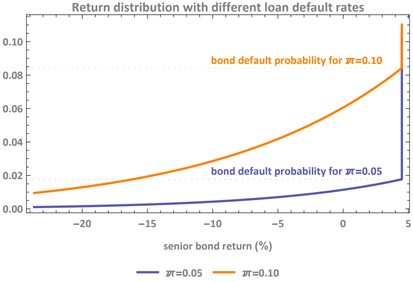
Risk analysis of senior bond

- Higher pool loss default rate and higher default correlation both bad for senior bond
  - I.e. shift return distribution function to left
- Risk of senior bond is very sensitive to default correlation
  - With correlation very low, senior bond default probability low even with high pool default rate
- High default correlation induces higher probability of default clusters that can reach into senior tranche
- Table displays default *probabilities* for different settings of pool loss distribution *parameters*

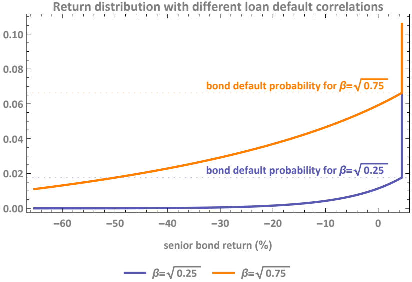
Senior bond default probability

	Correlation assumptions			
	low	moderate	high	very high
	$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$	0.0000	0.0031	0.0184	0.0309
$\pi = 0.05$	0.0000	0.0177	0.0503	0.0663
$\pi = 0.10$	0.0030	0.0842	0.1297	0.1390

Pool default behavior and senior bond returns



Both plots use baseline parameter $\beta = \sqrt{0.25}$.



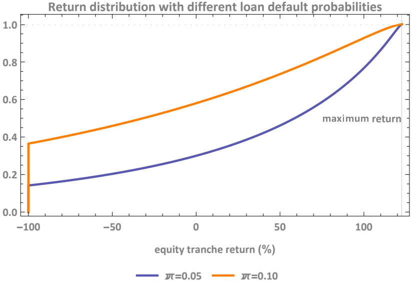
Both plots use baseline parameter $\pi = 0.05$.

Risk analysis of equity tranche

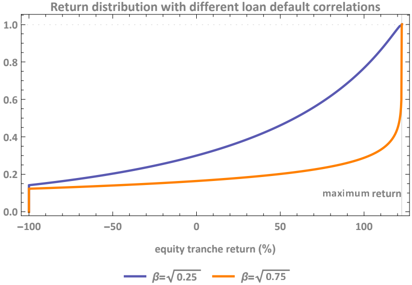
- Increase in pool default rate decreases equity returns
  - Return distribution shifts to the *left*
- High default correlation increases equity returns
  - Return distribution shifts to the *right*
  - Equity has limited downside but unlimited upside
  - High correlation→high likelihood of very many and very few defaults
  - Former doesn't diminish expected return, since equity tranche value cannot go below zero, but latter adds to expected return
- Table displays *probabilities* of a loss on the equity tranche for different settings of pool loss distribution *parameters*

Equity tranche: probability of negative return				
	Correlation assumptions			
	low	moderate	high	very high
	$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$	0.0355	0.1241	0.1200	0.0899
$\pi = 0.05$	0.3458	0.3000	0.2328	0.1642
$\pi = 0.10$	0.8903	0.5801	0.4146	0.2884

Pool default behavior and equity tranche returns



Both plots use baseline parameter $\beta = \sqrt{0.25}$.



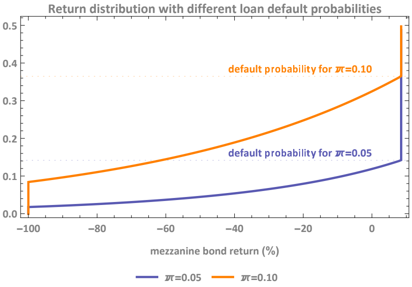
Both plots use baseline parameter $\pi = 0.05$.

Risk analysis of mezzanine tranche

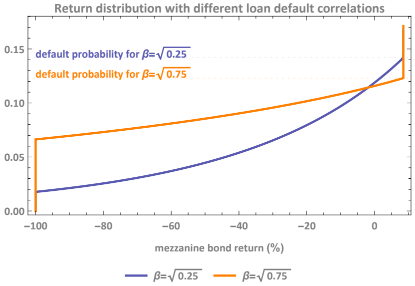
- Increase in pool default rate decreases return
- Impact of default correlation more ambiguous than for senior and equity
  - Will generally benefit less than equity and suffer less than senior from higher correlation
  - Depends heavily on attachment/detachment points
- Table displays default *probabilities* for different settings of pool loss distribution *parameters*

Mezzanine bond default probability				
	Correlation assumptions			
	low	moderate	high	very high
	$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$	0.0007	0.0443	0.0679	0.0638
$\pi = 0.05$	0.0379	0.1418	0.1478	0.1230
$\pi = 0.10$	0.4401	0.3648	0.2973	0.2295

Pool default behavior and mezzanine tranche returns



Both plots use baseline parameter $\beta = \sqrt{0.25}$.



Both plots use baseline parameter $\pi = 0.05$.

Risk modeling and structuring of securitizations

- Risk modeling used to structure a securitization
  - Attachment and detachment points, i.e. tranche sizes
  - Structure also affected by assessment of pool credit quality and cash flows
- **Example:** Suppose it is desired that senior bond have a default probability no greater than 1 percent
  - Find required attachment point a_s, given pool credit parameters
- Required attachment point satisfies

$$x^{\circ} = 1 - \frac{(1 - a_s)(1 + c_s)}{1 + r_l}$$
$$0.01 = 1 - \mathbf{P}\left[\tilde{x} \leq x^{\circ}\right]$$

- Using baseline parameters, required attachment point is $a_s = 0.7552$
- Equity and/or mezzanine tranches will need to be somewhat wider

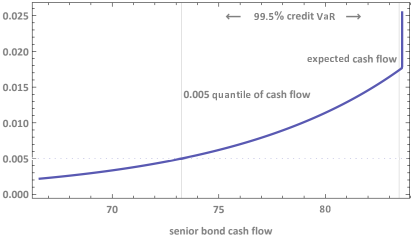
Ratings inflation

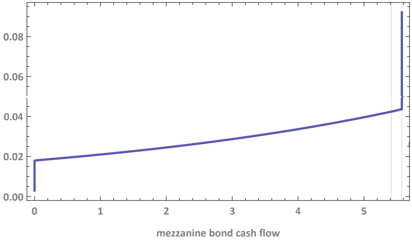
- **Ratings inflation:** assignment by rating agencies of unwarranted high ratings to bonds, particulalrly securitization tranches
- Can be achieved i.a. through
  - Setting attachment points so that senior tranches larger
  - Underestimate loan pool default probabilities, expected loss
- Motivation:
  - **"Issuer-pays":** rating agencies paid by issuers, who benefit from having lower-coupon senior bonds a larger share of liabilities
  - But investors also interested in higher ratings to satisfy regulatory constraints, increase available highly-rated issue volume

Credit Value-at-Risk of a securitization

- Credit VaR can be computed using two components
  - Quantile of cash flow or return to any tranche using loss distribution
  - Expected loss (EL) as integral of product of loss density and cash flow or return
- Credit VaR equal to loss at a specified quantile minus EL

Credit VaR of bond tranches





Vertical grid lines in each plot placed at cash flow with specified cumulative probability, and at expected value of cash flow. The distance between the two grid lines is the credit VaR for the tranche. Both plots use baseline parameters $\pi = 0.05$ and $\beta = \sqrt{0.25}$.

Tail risk

- Low default probability but very high LGD
- Clusters of default: what if more likely?
- Much higher default probability than assumed
- Combination of ratings and capital standards

Correlation and convexity

- Market risk consequences of tail risk
- The role of default correlation
- Equity-AAA tradeoff
    - High correlation benefits equity, reduces value of AAA
    - Low correlation: high probability of steady trickle of defaults, unless default probability very low